International Journal of Heat and Mass Transfer 52 (2009) 5792–5795

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid

D.A. Nield [a], A.V. Kuznetsov [b,*]

[a] Department of Engineering Science, University of Auckland, Private Bag 92019, Auckland 1142, New Zealand
[b] Department of Mechanical and Aerospace Engineering, North Carolina State University, Campus Box 7910, Raleigh, NC 27695-7910, USA

ARTICLE INFO

Article history:
Received 14 April 2009
Received in revised form 31 July 2009
Accepted 31 July 2009
Available online 11 September 2009

Keywords:
Cheng–Minkowycz problem
Natural convection
Nanofluid
Porous medium
Similarity solution

ABSTRACT

The Cheng–Minkowycz problem of natural convection past a vertical plate, in a porous medium saturated by a nanofluid, is studied analytically. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. For the porous medium the Darcy model is employed. A similarity solution is presented. This solution depends on a Lewis number Le, a buoyancy-ratio number Nr, a Brownian motion number Nb, and a thermophoresis number Nt. The dependency of the Nusslelt number on these four parameters is investigated.

1. Introduction

The term "nanofluid" refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles). The term was coined by Choi [1]. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [2]. This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems (Buongiorno and Hu [3]).

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [4], who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on the further heat transfer enhancement observed in convective situations. Buongiorno notes that several authors have suggested that convective heat transfer enhancement could be due to the dispersion of the suspended nanoparticles but he argues that this effect is too small to explain the observed enhancement. Buongiorno also concludes that turbulence is not affected by the presence of the nanoparticles so this cannot explain the observed enhancement. Particle rotation has also been proposed as a cause of heat transfer enhancement, but Buongiorno calculates that this effect is too small to explain the effect. With dispersion, turbulence and particle rotation ruled out as significant agencies for heat transfer enhancement, Buongiorno proposed a new model based on the mechanics of the nanoparticle/base-fluid relative velocity.

Buongiorno [4] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (that he calls the slip velocity). He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling. After examining each of these in turn, he concluded that in the absence of turbulent effects it is the Brownian diffusion and the thermophoresis that will be important. Buongiorno proceeded to write down conservation equations based on these two effects.

The problem of natural convection in a porous medium past a vertical plate is a classical problem first studied by Cheng and Minkowycz [5]. The problem is presented as a paradigmatic configuration and solution in the book by Bejan [6]. The extension to the case of heat and mass transfer was made by Bejan and Khair [7]. Further work on this topic is surveyed in Sections 5.1 and 9.2.1 in Nield and Bejan [8]. A review of the heat transfer characteristics of nanofluids has been made by Wang and Mujumdar [9].

In the present paper the model of [4] is applied to the problem in [5].

2. Analysis

It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix.

* Corresponding author.
E-mail addresses: d.nield@auckland.ac.nz (D.A. Nield), avkuznet@eos.ncsu.edu (A.V. Kuznetsov).

0017-9310/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2009.07.024

Nomenclature

D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	rescaled nanoparticle volume fraction, defined by Eq. (20)
$\mathbf{g}$	gravitational acceleration vector
k_m	effective thermal conductivity of the porous medium
K	permeability of the porous medium
Le	Lewis number, defined by Eq. (27)
Nr	buoyancy ratio, defined by Eq. (24)
Nb	Brownian motion parameter, defined by Eq. (25)
Nt	thermophoresis parameter, defined by Eq. (26)
Nu	Nusselt number, defined by Eq. (32)
Nur	reduced Nusselt number, $\mathrm{Nu}/\mathrm{Ra}_x^{1/2}$
p	pressure
q''	wall heat flux
Ra_x	local Rayleigh number, defined by Eq. (18)
s	dimensionless stream function, defined by Eq. (20)
T	temperature
T_w	temperature at the vertical plate
T_∞	ambient temperature attained as y tends to infinity
$\mathbf{v}$	Darcy velocity, (u, v)
(x, y)	Cartesian coordinates (x-axis is aligned vertically upwards, plate is at $y = 0$)

Greek symbols

α_m	thermal diffusivity of the porous medium, $\frac{k_m}{(\rho c)_f}$
β	volumetric expansion coefficient of the fluid
ε	porosity
η	similarity variable, defined by Eq. (19)
θ	dimensionless temperature, defined by Eq. (20)
μ	viscosity of the fluid
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of the porous medium
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	parameter defined by Eq. (13), $\frac{\varepsilon(\rho c)_p}{(\rho c)_f}$
ϕ	nanoparticle volume fraction
ϕ_w	nanoparticle volume fraction at the vertical plate
ϕ_∞	ambient nanoparticle volume fraction attained as y tends to infinity
ψ	stream function, defined by Eq. (14)

We consider a two-dimensional problem. We select a coordinate frame in which the x-axis is aligned vertically upwards. We consider a vertical plate at $y = 0$. At this boundary the temperature T and the nanoparticle fraction ϕ take constant values T_w and ϕ_w, respectively. The ambient values, attained as y tends to infinity, of T and ϕ are denoted by T_∞ and ϕ_∞, respectively.

The Oberbeck-Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is assumed. We consider a porous medium whose porosity is denoted by ε and permeability by K. The Darcy velocity is denoted by $\mathbf{v}$. The following four field equations embody the conservation of total mass, momentum, thermal energy, and nanoparticles, respectively. The field variables are the Darcy velocity $\mathbf{v}$, the temperature T and the nanoparticle volume fraction ϕ.

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{v}}{\partial t} = -\nabla p - \frac{\mu}{K}\mathbf{v} + [\phi\rho_p + (1-\phi)\{\rho_f(1-\beta(T-T_\infty))\}]\mathbf{g}, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f \mathbf{v}\cdot\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_p[D_B\nabla\phi\cdot\nabla T + (D_T/T_\infty)\nabla T\cdot\nabla T], \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v}\cdot\nabla\phi = D_B\nabla^2\phi + (D_T/T_\infty)\nabla^2 T. \tag{4}$$

We write $\mathbf{v} = (u, v)$.

Here ρ_f, μ and β are the density, viscosity, and volumetric volume expansion coefficient of the fluid while ρ_P is the density of the particles. The gravitational acceleration is denoted by $\mathbf{g}$. We have introduced the effective heat capacity $(\rho c)_m$, and the effective thermal conductivity k_m of the porous medium. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T. Details of the derivation of Eqs. (3) and (4) are given in the papers by Buongiorno [4], Tzou [10,11] and Nield and Kuznetsov [12,13]. The flow is assumed to be slow so that an advective term and a Forchheimer quadratic drag term do not appear in the momentum equation.

The boundary conditions are taken to be

$$v = 0, \quad T = T_w, \phi = \phi_w \text{ at } y = 0, \tag{5}$$

$$u = v = 0, \quad T \to T_\infty, \phi \to \phi_\infty \text{ as } y \to \infty. \tag{6}$$

We consider a steady state flow.

In keeping with the Oberbeck-Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, we can linearize the momentum equation and write Eq. (2) as

$$0 = -\nabla p - \frac{\mu}{K}\mathbf{v} + [(\rho_p - \rho_{f\infty})(\phi - \phi_\infty) + (1-\phi_\infty)\rho_{f\infty}\beta(T-T_\infty)]\mathbf{g}. \tag{7}$$

We now make the standard boundary-layer approximation, based on a scale analysis, and write the governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K}u + \left[(1-\phi_\infty)\rho_{f\infty}\beta g(T-T_\infty) - (\rho_p - \rho_{f\infty})g(\phi-\phi_\infty)\right] \tag{9}$$

$$\frac{\partial p}{\partial y} = 0, \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\nabla^2 T + \tau\left[D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{11}$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y}\right) = D_B\frac{\partial^2\phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \tag{12}$$

where

$$\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \tau = \frac{\varepsilon(\rho c)_p}{(\rho c)_f}. \tag{13}$$

One can eliminate p from Eqs. (9) and (10) by cross-differentiation. At the same time one can introduce a stream function ψ defined by

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}, \tag{14}$$

so that Eq. (8) is satisfied identically.We are then left with the following three equations.

$$\frac{\partial^2\psi}{\partial y^2} = \frac{(1-\phi_\infty)\rho_{f\infty}\beta gK}{\mu}\frac{\partial T}{\partial y} - \frac{(\rho_p - \rho_{f\infty})gK}{\mu}\frac{\partial\phi}{\partial y} \tag{15}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \alpha_m\nabla^2 T + \tau\left[D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{16}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right) = D_B\frac{\partial^2\phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \tag{17}$$

We now introduce the local Rayleigh number Ra_x defined by

$$\mathrm{Ra}_x = \frac{(1-\phi_\infty)\rho_{f\infty}\beta g K x}{\mu\alpha_m}, \tag{18}$$

and the similarity variable

$$\eta = \frac{y}{x}\mathrm{Ra}_x^{1/2}. \tag{19}$$

This choice is made on the basis of scale analysis (see Section 9.2.1 of Nield and Bejan [8]). Since most nanofluids examined to date have large values for the Lewis number, we are interested mainly in the case Le > 1. Also we are interested in the case where it is heat transfer (rather than mass transfer) that is driving the flow. In the present context this means that we are assuming that the buoyancy-ratio parameter Nr defined by Eq. (24) below is small compared with unity and that the Lewis number Le defined by Eq. (27) is larger than unity. The reader is referred to Figure 9.5 and Table 9.1 in [8], which are based on Figure 2 and Table 1 in [7].

We also introduce the dimensionless variables s, θ, and f defined by

$$s(\eta) = \frac{\psi}{\alpha_m \mathrm{Ra}_x^{1/2}}, \quad \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad f(\eta) = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}. \tag{20}$$

Then, on substitution in Eqs. (15)–(17), we obtain the ordinary differential equations

$$s'' - \theta' + \mathrm{Nr} f' = 0, \tag{21}$$

$$\theta'' + \frac{1}{2}s\theta' + \mathrm{Nb} f'\theta' + \mathrm{Nt}\theta'^2 = 0, \tag{22}$$

$$f'' + \frac{1}{2}\mathrm{Le} s f' + \frac{\mathrm{Nt}}{\mathrm{Nb}}\theta'' = 0, \tag{23}$$

where the four parameters are defined by

$$\mathrm{Nr} = \frac{(\rho_p - \rho_{f\infty})(\phi_w - \phi_\infty)}{\rho_{f\infty}\beta(T_w - T_\infty)(1-\phi_\infty)}, \tag{24}$$

$$\mathrm{Nb} = \frac{\varepsilon(\rho c)_p D_B(\phi_w - \phi_\infty)}{(\rho c)_f \alpha_m}, \tag{25}$$

$$\mathrm{Nt} = \frac{\varepsilon(\rho c)_p D_T(T_w - T_\infty)}{(\rho c)_f \alpha_m T_\infty}, \tag{26}$$

$$\mathrm{Le} = \frac{\alpha_m}{\varepsilon D_B}. \tag{27}$$

Here Nr, Nb, Nt, Le denote a buoyancy ratio, a Brownian motion parameter, a thermophoresis parameter, and a Lewis number, respectively.

Equations (21)–(23) are solved subject to the following boundary conditions:

$$\text{At } \eta = 0 : s = 0, \quad \theta = 1, \quad f = 1. \tag{28}$$

$$\text{As } \eta \to \infty : \quad s' = 0, \quad \theta = 0, \; f = 0. \tag{29}$$

Integrating Eq. (21) once and using boundary conditions (29) results in

$$s' - \theta + \mathrm{Nr} f = 0, \tag{30}$$

Equations (30), (22), and (23) are solved subject to boundary conditions at $\eta = 0$ given by Eq. (28) and the following boundary conditions at $\eta \to \infty$:

$$\text{At}\eta \to \infty : \theta = 0, \quad f = 0. \tag{31}$$

When Nr, Nb and Nt are all zero, Eqs. (21) and (22) involve just two dependent variables, namely s and θ, and the boundary-value problem for these two variables reduces to the classical problem solved by Cheng and Minkowycz [5]. (The boundary-value problem for f then becomes ill-posed and is of no physical significance.)

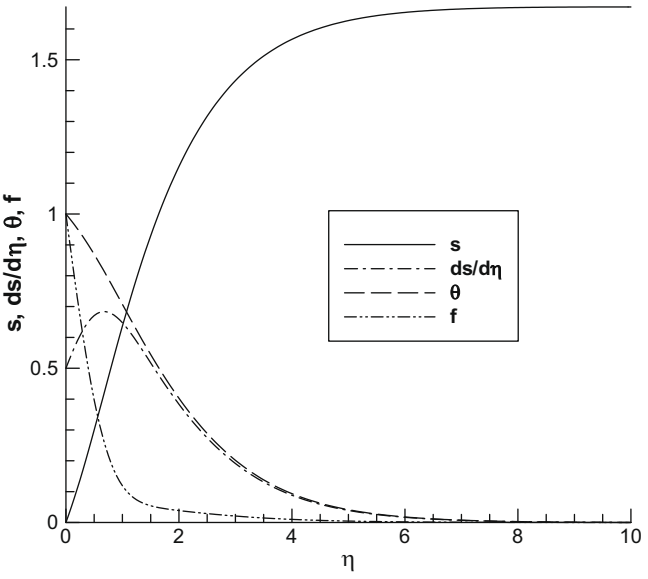

Fig. 1. Plots of dimensionless similarity functions $s(\eta)$, $s'(\eta)\theta(\eta)$, $f(\eta)$ for the case Le = 10, Nr = 0.5, Nb = 0.5, Nt = 0.5.

A quantity of practical interest is the Nusselt number Nu defined by

$$\mathrm{Nu} = \frac{q'' x}{k_m(T_w - T_\infty)}, \tag{32}$$

where q'' is the wall heat flux and k_m is the effective thermal conductivity of the porous medium. In the present context $\mathrm{Nu}/\mathrm{Ra}_x^{1/2}$ is represented by $-\theta'(0)$. (Likewise the dimensionless mass flux is represented by a Sherwood number Sh which is proportional to $-f'(0)$, but this of lesser interest here.)

3. Results and discussion

Plots of the similarity variables for a typical case, chosen as that for Le = 10, Nr = 0.5, Nb = 0.5, Nt = 0.5, are shown in Fig. 1. The boundary-layer profiles for the temperature function $\theta(\eta)$ and the stream function $s(\eta)$ have essentially the same form as in the case of a regular fluid. The thickness of the boundary-layer for the mass fraction function $f(\eta)$ is smaller than the thermal boundary-layer thickness when Le > 1. It is well known that, in the case of a regular fluid, the profile for $ds/d\eta$ (something that represents the longitudinal component of the velocity, u) is identical with that for the temperature θ. We now see that in the case of a nanofluid the two profiles diverge within a layer whose thickness is comparable with that of the mass fraction.

Table 1
Linear regression coefficients and error bound for the reduced Nusselt number. Here C_r, C_b, C_t, are the coefficients in the linear regression estimate $\mathrm{Nu}_{est}/\mathrm{Ra}_x^{1/2} = 0.444 + C_r\mathrm{Nr} + C_b\mathrm{Nb} + C_t\mathrm{Nt}$, and $\hat{\varepsilon}$ is the maximum relative error defined by $\hat{\varepsilon} = |(\mathrm{Nu}_{est} - \mathrm{Nu})/\mathrm{Nu}|$, applicable for Nr, Nb, Nt each in [0, 0.5].

Le	C_r	C_b	C_t	$\hat{\varepsilon}$
1	−0.309	−0.060	−0.166	0.154
2	−0.230	−0.129	−0.162	0.147
5	−0.148	−0.209	−0.152	0.126
10	−0.111	−0.245	−0.150	0.119
20	−0.086	−0.268	−0.149	0.114
50	−0.064	−0.288	−0.149	0.110
100	−0.053	−0.298	−0.148	0.108
200	−0.045	−0.304	−0.148	0.107
500	−0.039	−0.310	−0.148	0.106
1000	−0.036	−0.313	−0.148	0.107

Table 2
Quadratic regression coefficients and error bound for the reduced Nusselt number. Here C_{r1}, C_{r2}, C_{b1}, C_{b2}, C_{t1}, C_{t2}, C_{bt}, C_{tr}, and C_{rb} are the coefficients in the quadratic regression estimate $\mathrm{Nu}_{est}/\mathrm{Ra}_x^{1/2} = 0.444 + C_{r1}\mathrm{Nr} + C_{b1}\mathrm{Nb} + C_{t1}\mathrm{Nt} + C_{r2}\mathrm{Nr}^2 + C_{b2}\mathrm{Nb}^2 + C_{t2}\mathrm{Nt}^2 + C_{bt}\mathrm{NbNt} + C_{tr}\mathrm{NtNr} + C_{rb}\mathrm{NrNb}$, and $\hat{\varepsilon}$ is the maximum relative error defined by $\hat{\varepsilon} = |(\mathrm{Nu}_{est} - \mathrm{Nu})/\mathrm{Nu}|$, applicable for Nr, Nb, Nt each in [0, 0.5].

Le	C_{r1}	C_{b1}	C_{t1}	C_{r2}	C_{b2}	C_{t2}	C_{bt}	C_{tr}	C_{rb}	$\hat{\varepsilon}$
1	−0.353	−0.020	−0.223	−0.040	−0.381	0.067	0.284	−0.122	0.451	0.066
2	−0.249	−0.130	−0.206	−0.066	−0.279	0.040	0.268	−0.102	0.397	0.100
5	−0.160	−0.251	−0.192	−0.028	−0.064	0.042	0.162	−0.014	0.206	0.035
10	−0.115	−0.306	−0.189	−0.016	0.017	0.042	0.135	0.007	0.134	0.014
20	−0.083	−0.344	−0.187	−0.009	0.065	0.041	0.124	0.0135	0.091	0.007
50	−0.055	−0.375	−0.186	−0.004	0.103	0.041	0.121	0.014	0.056	0.007
100	−0.041	−0.390	−0.186	−0.001	0.119	0.041	0.121	0.013	0.040	0.006
200	−0.031	−0.400	−0.185	0.001	0.130	0.041	0.121	0.012	0.029	0.006
500	−0.022	−0.409	−0.185	0.003	0.139	0.041	0.122	0.010	0.020	0.006
1000	−0.018	−0.413	−0.185	0.004	0.143	0.041	0.123	0.009	0.016	0.006

For the case Le = 10, the value of $\mathrm{Nu}/\mathrm{Ra}_x^{1/2}$ (something that we will refer to as the reduced Nusselt number and denote by Nur) was calculated for 125 sets of values of Nr, Nb, Nt in the range [0.1, 0.2, 0.3, 0.4, 0.5] and a linear regression was performed on the results. This yielded the correlation

$$\mathrm{Nur}_{est} = 0.444 - 0.111\mathrm{Nr} - 0.245\mathrm{Nb} - 0.150\mathrm{Nt}, \tag{33}$$

valid for Nr, Nb, Nt each taking values in the range [0, 0.5], with a maximum error of about 12%. Clearly an increase in any of the buoyancy-ratio number Nr, the Brownian motion parameter Nb, or the thermophoresis parameter Nt leads to a decrease in the value of the reduced Nusselt number (corresponding to an increase in the thermal boundary-layer thickness). The maximum error occurs at the extreme end of the range considered, namely when (Nr, Nb, Nt) = (0.5, 0.5, 0.5), and the correlation formula overestimates the reduction from the standard fluid value 0.444.

This exercise was repeated for other values of Le, with the results shown in Table 1.

These results show that the coefficient of Nt varies little as Le varies. This result is to be expected since, from the form of Eq. (23), one can anticipate that when Le is large the variable f' will decay rapidly as η increases, and then since the term in Nt in Eq. (22) does not involve f one can anticipate that the contribution from Nt will not depend markedly on the value of Le. On the other hand, the coefficient of Nr decreases markedly as Le increase while the coefficient of Nt changes in the other direction. Finally from Table 1 we observe that the accuracy of the linear regression estimate increases as Le increases.

We believe that for most practical purposes the simple linear regression formula in Eq. (33) should be adequate. If one wants a more accurate formula then one can perform a quadratic regression. For the case Le = 10, for example, we obtained instead of Eq. (33) the formula

$$\begin{aligned}\mathrm{Nur}_{est} = {} & 0.444 - 0.115\mathrm{Nr} - 0.306\mathrm{Nb} - 0.189\mathrm{Nt} - 0.016\mathrm{Nr}^2 \\ & + 0.017\mathrm{Nb}^2 + 0.042\mathrm{Nt}^2 + 0.135\mathrm{NbNt} + 0.007\mathrm{NtNr} \\ & + 0.134\mathrm{NrNb},\end{aligned} \tag{34}$$

which gives a maximum error of just 1.4% over the same range. The relatively large interactions between Nr and Nb (displayed by the coefficient of the last term in Eq. (34)), and between Nr and Nt (displayed by the coefficient of the third to last term in Eq. (34)) are of interest. Values of the coefficients for some other cases are presented in Table 2.

4. Conclusions

We have examined the influence of nanoparticles on natural convection boundary-layer flow in a porous medium past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. In this pioneering study we have employed the Darcy model for the momentum equation and we have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. This permits a simple similarity solution which depends on four dimensionless parameters, namely a Lewis number Le, a buoyancy-ratio parameter Nr, a Brownian motion parameter Nb, and a thermophoresis parameter Nt. We have explored the way in which the wall heat flux, represented by a Nusselt number Nu, depends on these four parameters. Since we are dealing with the case of convection driven mainly by heat transfer we expect that the boundary condition on the nanoparticle fraction is of lesser importance.

References

[1] S. Choi, Enhancing thermal conductivity of fluids with nanoparticle, in: D.A. Siginer, H.P. Wang, (Eds.), Developments and Applications of Non-Newtonian Flows, ASME FED – vol. 231, MD – vol. 66, 1995, pp. 99–105.
[2] H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles, Netsu Bussei 7 (1993) 227–233.
[3] J. Buongiorno, W. Hu, Nanofluid coolants for advanced nuclear power plants, in: Proceedings of ICAPP '05, Paper no. 5705, Seoul, May 15–19, 2005.
[4] J. Buongiorno, Convective transport in nanofluids, ASME J. Heat Transfer 128 (2006) 240–250.
[5] P. Cheng, W.J. Minkowycz, Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike, J. Geophys. Res. 82 (1977) 2040–2044.
[6] A. Bejan, Convection Heat Transfer, third ed., Wiley, Hoboken, 2004. pp. 586–591.
[7] A. Bejan, K.R. Khair, Heat and mass transfer by natural convection in a porous medium, Int. J. Heat Mass Transfer 28 (1985) 909–919.
[8] D.A. Nield, A. Bejan, Convection in Porous Media, third ed., V Springer, New York, 2006.
[9] X.Q. Wang, A.S. Mujumdar, Heat transfer characteristics of nanofluids: a review, Int. J. Thermal Sci. 46 (2007) 1–19.
[10] D.Y. Tzou, Instability of nanofluids in natural convection, ASME J. Heat Transfer 130 (2008) #072401.
[11] D.Y. Tzou, Thermal instability of nanofluids in natural convection, Int. J. Heat Mass Transfer 51 (2008) 2967–2979.
[12] D.A. Nield, A.V. Kuznetsov, The onset of convection in a nanofluid layer, ASME J. Heat Transfer, submitted for publication.
[13] D.A. Nield, A.V. Kuznetsov, Thermal instability in a porous medium layer saturated by a nanofluid, Int. J. Heat Mass Transfer 52 (2009) 5796–5801.